Testing a mathematical derivation of climate sensitivity

I aim to show that energy balance models which assume high equilibrium sensitivity (A) like the ones in the text overstate the sensitivity of the current climate state by a factor of 3-4 over an empirically derived sensitivity and that the sensitivity given below at B is more appropriate for the current CAGW debate.

Climate sensitivity to insolation changes appears to be 0.8 +/- 0.2 under ice-albedo feedback, and exhibits highest sensitivity at the positive going transition stage. This sensitivity would give a temp rise in line with IPCC lower estimates.

$\Delta T=\Delta F*\lambda$ = 3.7*0.8 +/-0.2 of ΔT = 2.95 +/- 0.8 °C with a range 2.2 – 3.7 °C (A)

Conversely temperatures above the **Tref = 0°C Vostok** value indicate low sensitivity and greater stability with an inferred sensitivity of **λ = 0.25 +/-0.05 (B)**

This would give a temp rise of **0.8°C +/- 0.2°C** for a further doubling of CO_2 to 800ppm

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Testing a mathematical derivation of climate sensitivity

The following bifurcation diagram is taken from

http://research.atmos.ucla.edu/tcd//PREPRINTS/Math_clim-Taipei-M_Ghil_vf.pdf

A Mathematical Theory of Climate Sensitivity or, How to Deal With Both Anthropogenic Forcing and Natural Variability? By Michael Ghil UCLA

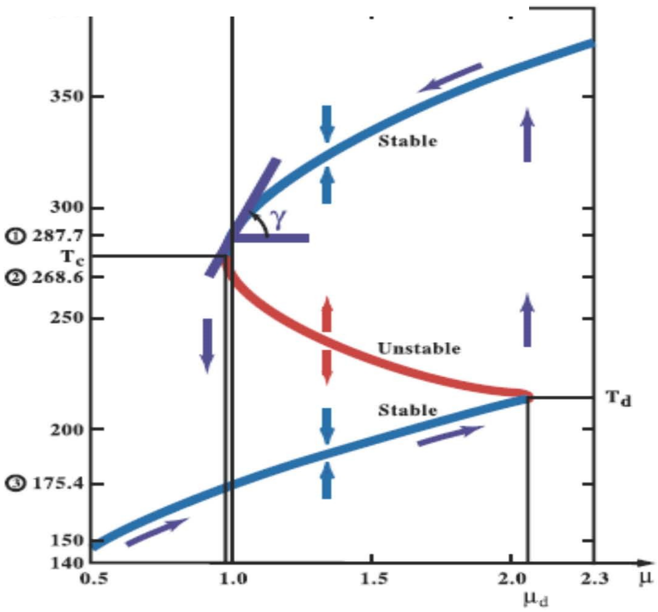
Notes from original:

The bifurcation diagram of a 1-D EBM, like the one of Eq. (1.2), is shown in Fig. 1.1. It displays the model's mean temperature T as a function of the fractional change in the insolation Q = Q(x) at the top of the atmosphere.

The S-shaped curve in the figure arises from two back-to-back saddle-node bifurcations. model for the evolution of surface-air temperature T = T(x; t), say,

$$c(x)\frac{\partial \bar{T}}{\partial t} = R_i - R_o + D. \qquad (1.2)$$

Fig. 1.1. Bifurcation diagram for the solutions of an energy-balance model (EBM), showing the global-mean temperature $\bar{T}$ vs. the fractional change μ of insolation at the top of the atmosphere. The arrows pointing up and down at about $\mu = 1.4$ indicate the stability of the branches: towards a given branch if it is stable and away if it is unstable. The other arrows show the hysteresis cycle that global temperatures would have to undergo for transition from the upper stable branch to the lower one and back. The angle γ gives the measure of the present climates sensitivity to changes in insolation. [After Ghil and Childress (1987) with permission from Springer-Verlag.]



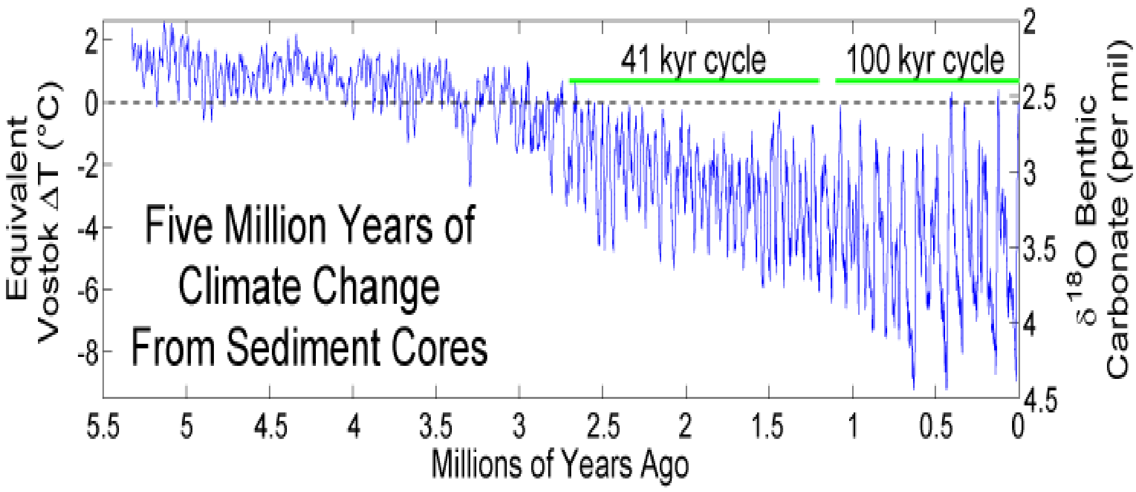
I have extracted the following info from the diagram

Sensitivity λ = tan(γ) for unit insolation change in the range of λ = tan(60) = 1.7 tan(45) = 1. tan(30) = 0.58, tan(20)= 0.36

Current temperature T = 287.7°K and its mirror value about the bifurcation T = 268.6°K
Bifurcation temperature Tc = 278.15°K, low stable temperature(ice-house) Td = 220°K

Testing Climate Sensitivity 1

I will show an empirical derivation of climate sensitivity from the following published temperature graph.



"Vostok 420ky 4curves insolation". Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Vostok_420ky_4curves_insolation.jpg#/media/File:Vostok_420ky_4curves_insolation.jpg

The graph is a paleo-temperature reconstruction published on wikipedia

The following are the basic parameters I will use:

Tref is 0°C Vostok Eq the current baseline
climate sensitivity can be derived from λ =ΔT/ΔF
Solar forcing is 7.1 W/m2
Forcing from a doubling of CO_2 is 3.7 W/m2 from IPCC Tar

The temperature is assumed to be in equilibrium and temperature change initially dependant on solar forcing (change in insolation) the following table is for the average temperature of the period

at T = Tref +2°C, λ =ΔT/ΔF = 1.5/7.1= 0.21 °C/(Wm−2) = tan(12) or =tan(23)/2
at T = Tref +1°C, λ =ΔT/ΔF = 2/7.1 = 0.28 °C/(Wm−2) = tan(16) or =tan(29)/2
at T = Tref, λ =ΔT/ΔF = 2.5/7.1 = 0.35 °C/(Wm−2) = tan(20) or =tan(35)/2
at T = Tref -1°C, λ =ΔT/ΔF = 3/7.1 = 0.42 °C/(Wm−2) = tan(23) or =tan(39)/2
at T = Tref -2°C, λ =ΔT/ΔF = 4/7.1 = 0.56 °C/(Wm−2) = tan(29) or =tan(49)/2
at T = Tref -3°C, λ =ΔT/ΔF = 4/7.1 = 0.56 °C/(Wm−2) = tan(29) or =tan(49)/2
at T = Tref -4°C, λ =ΔT/ΔF = 4/7.1 = 0.56 °C/(Wm−2) = tan(29) or =tan(49)/2
at T = Tref -5°C, λ =ΔT/ΔF = 10/7.1 = 1.4 °C/(Wm−2) = tan(55) or =tan(70)/2 positive direction only
at T = Tref -6°C, λ =ΔT/ΔF = 3/7.1 = 0.42 °C/(Wm−2) = tan(23) or =tan(39)/2 from graph 2

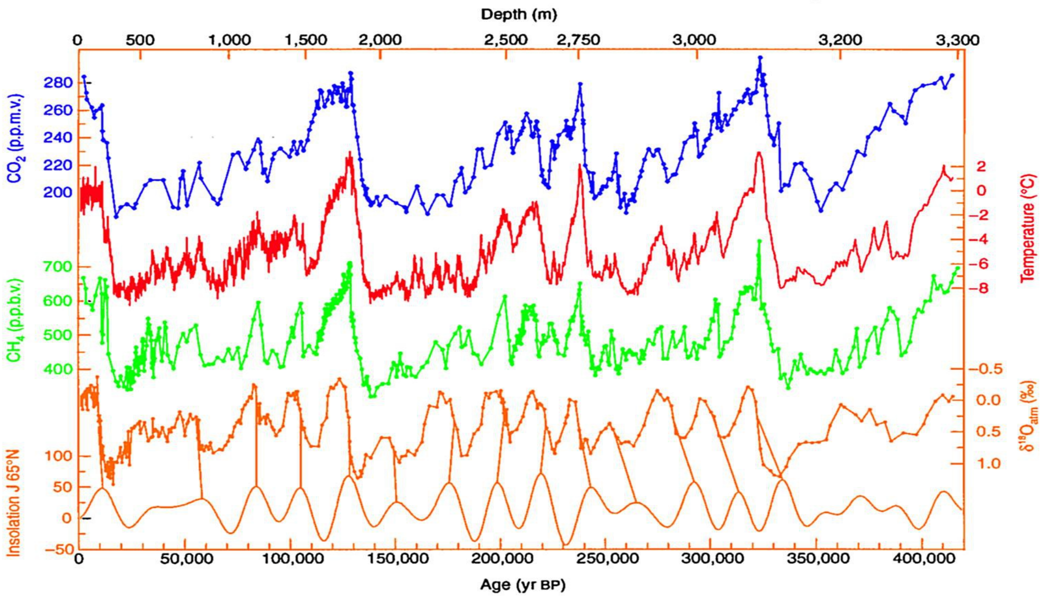
It can be seen that the forcing is temperature dependant and the increasing trend is theoretically a function of positive Ice feedback. A rate change occurs at at T = Tref -5°C

For rising temperature and CO_2 it is indicated that equilibrium ΔT is in the order of 0.35*3.7 (1.3°C) tending to 0.28*3.7 (1.03°C) and a further doubling tending to 0.21*3.7 (0.78°C). I have considered the action of feedbacks and as the graph tends to lower sensitivity with rising temperature then the feedbacks will be net negative for a positive forcing further constraining temperature rises.

The sensitivity extrapolation can be refined using the following graph which indicates a sensitivity after T = Tref -2°C of λ = 0.4 +/- 0.1 °C/(Wm–2) and that the sensitivity may be dependant on the direction of the temperature trend. Such that between Tref -5°C and Tref -2°C when warming a 'tippimg point' occurs indicating some hysteresis and a quasi bi-stable function The equilibrium sensitivity appears lower than the mathematically derived sensitivity for the current era by a **factor >2**.

The empirically derived sensitivity from insolation forcing is:

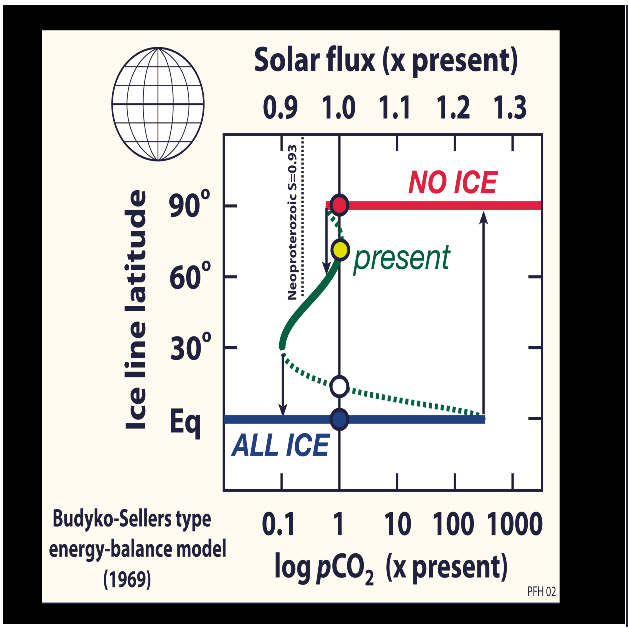
λ =ΔT/ΔF = 0.28 +/- 0.07 and ΔT = 1°C +/- 0.3°C temp rise for CO2 doubling

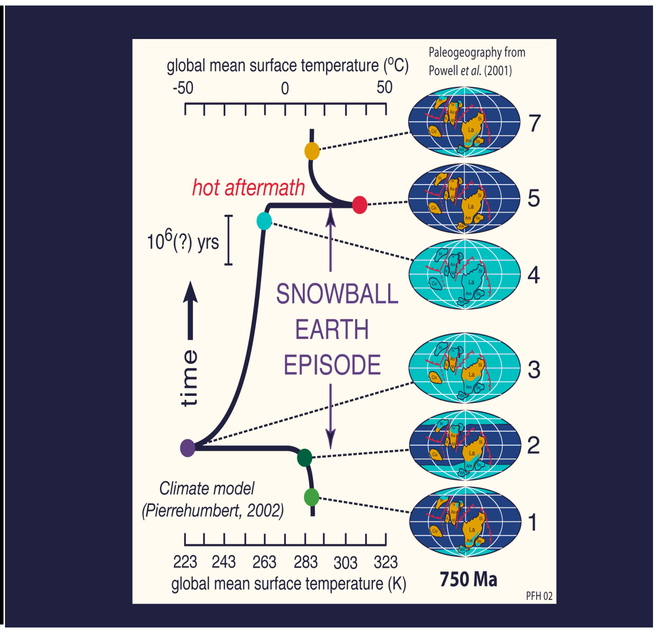


https://upload.wikimedia.org/wikipedia/commons/7/77/Vostok_420ky_4curves_insolation.jpg

Testing Climate Sensitivity 2

Teaching slides from Snowballearth.org http://www.snowballearth.org/slides.html





Testing sensitivity hypothesis on snowball earth. The energy balance model used here, gives a pCO_2 figure of 350 times current, or a doubling of between 8 and 9 over pre industrial levels (300 ppm).

I then derive forcing of between 8*3.7 and 9*3.7 W/m2 giving 29.6 and 33.3 (Wm^{-2}) for CO_2 concentration change. With $\lambda = 0.42$ then $\Delta T=\Delta F*\lambda$ which gives a range of 13 +/- 2**°C**.

But cyclical solar forcing of $\Delta F = 7.1*0.93 = 6.6$ gives $\Delta T = 6.6*0.42 = 2.8$ °C giving a maximum change of 18°C.

For $\lambda = 0.56$ the numbers are 17 +/- 2**°C** plus 3.7**°C** giving a maximum change of 23°C neither of which are enough to overcome the hysteresis. So the above derived sensitivities are too low

Assuming a total forcing of 48 +/ - 2 (Wm^{-2}) and a ΔT of 30,40,50°C gives a sensitivity range $\Delta T/\Delta F=\lambda$ of 0.68, 0.83, 1.0, equivalent to tan(34), tan(40), tan(45), which is in broad agreement with the model.

Assuming a total forcing of 48 +/ - 2 (Wm^{-2}) and a ΔT of 60-80 **°C** then $\Delta T/\Delta F=\lambda$ of 1.25-1.66 instantaneous sensitivity at the tipping point, this assumes a probable bifurcation between 270 and 290**°K**

Conclusion

This non rigorous hypothesis seems to show that climate sensitivity is highly dependant on the ice state with a tipping point behaviour on the post glacial transitions. Climate sensitivity to insolation changes appears to be 0.8 +/- 0.2 under ice-albedo feedback, and exhibits high sensitivity at the positive going transition stage. This sensitivity would give a temp rise in line with IPCC lower estimates.

$\Delta T=\Delta F*\lambda$ = 3.7*0.8 +/-0.2 of 2.95 +/- 0.8 °C range 2.2 – 3.7 °C

Conversely temperatures above the ref value indicate low sensitivity and greater stability with an inferred sensitivity of $\lambda = 0.25$ +/-0.05 this would give a temp rise of 0.8°C +/- 0.2°C for a further doubling of CO_2 to 800ppm

The bifurcation diagram shows general agreement with my arguments above both in absolute temperature values of the bifurcations, implied stability but the derived sensitivity from tangent of the angle γ for millenial stability **without ice feedback** is too high by a factor of 3 - 4 over the empirically derived value and that the governing equation contains ice albedo feedback parameters which are largely missing above Tref = 0°C Vostok Eq

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